

Home Work 1

$$\alpha_1 \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \alpha_3 \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{cases} \alpha_1 + 2\alpha_2 + 3\alpha_3 = 0 \\ -3\alpha_1 + 3\alpha_2 + 0\alpha_3 = 0 \\ 2\alpha_1 + 1\alpha_2 + 3\alpha_3 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 3 & 0 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Show that there exist } \alpha_1, \alpha_2, \alpha_3$$

Solution

$$\begin{array}{ccc|ccc} \alpha^T & & & I & & \\ \hline 1 & -3 & 2 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 3 & 0 & 3 & 0 & 0 & 1 \end{array}$$

$$\text{Row Operation} \rightarrow \begin{bmatrix} - & - & - & | & - & - & - \\ 0 & 9 & -3 & | & -2 & 1 & 0 \\ 0 & 9 & -3 & | & -3 & 0 & 1 \end{bmatrix}$$

$$\text{Row Operation} \rightarrow \begin{bmatrix} - & - & - & | & - & - & - \\ - & - & - & | & - & - & - \\ 0 & 0 & 0 & | & 1 & 1 & -1 \end{bmatrix}$$

$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = -1$ และเป็น Linearly Independent

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