HW Chapter 6

6.3. The data corresponding to sample 8 in Table 6.1 seem unusually large. Remove sample 8. Construct a joint 95% confidence region for the mean difference vector δ and the 95% Bonferroni simultaneous intervals for the components of the mean difference vector. Are the results consistent with a test of $H_o : \delta = 0$? Discuss. Does the "outlier" make a difference in the analysis of these data?

	Commercial lab		State lab of hygiene			
Sample	BOD	SS	BOD	SS	dj1	dj2
1	6	27	25	15	-19	12
2	6	23	28	13	-22	10
3	18	64	36	22	-18	42
4	8	44	35	29	-27	15
5	11	30	15	31	-4	-1
6	34	75	44	64	-10	11
7	28	26	42	30	-14	-4
9	43	54	34	56	9	-2
10	33	30	29	20	4	10
11	20	14	39	21	-19	-7
average					-12	8.6
Variance of d					136.4444	198.2667
Co-Variance of d					-52.44444444	

Construct a joint 95% confidence region for the mean difference vector δ

A 100(1-α)% confidence region for δ consists of all δ such that

$$(\bar{d}-\delta)'S_d^{-1}(\bar{d}-\delta) \le \frac{(n-1)p}{n(n-p)}F_{p,n-p}(\alpha)$$

$$= \begin{bmatrix} -12 & 8.6 \end{bmatrix} \begin{bmatrix} 0.008158 & 0.002158 \\ 0.002158 & 0.005615 \end{bmatrix} \begin{bmatrix} -12 \\ 8.6 \end{bmatrix}$$

$$\begin{bmatrix} -0.07934 & 0.022389 \end{bmatrix} \begin{bmatrix} -12 \\ 8.6 \end{bmatrix}$$

$= 1.144652$

$$\frac{(n-1)p}{n(n-p)}F_{p,n-p}(\alpha) = 1.001748$$

$$(\bar{d}-\delta)'S_d^{-1}(\bar{d}-\delta) = 1.144652 > \frac{(n-1)p}{n(n-p)}F_{p,n-p}(\alpha) = 1.001748$$

หรือ $T^2 = n(\bar{d}-\delta)'S_d^{-1}(\bar{d}-\delta) \le \frac{(n-1)p}{(n-p)}F_{p,n-p}(\alpha)$

$T^2 = 11.44652 > 3.167807$ **We reject H_o and conclude that there is a nonzero mean difference between the measurements of the two laboratories.**

หา Eigenvalues $\lambda_1 = 228.2318, \lambda_2 = 106.4793$

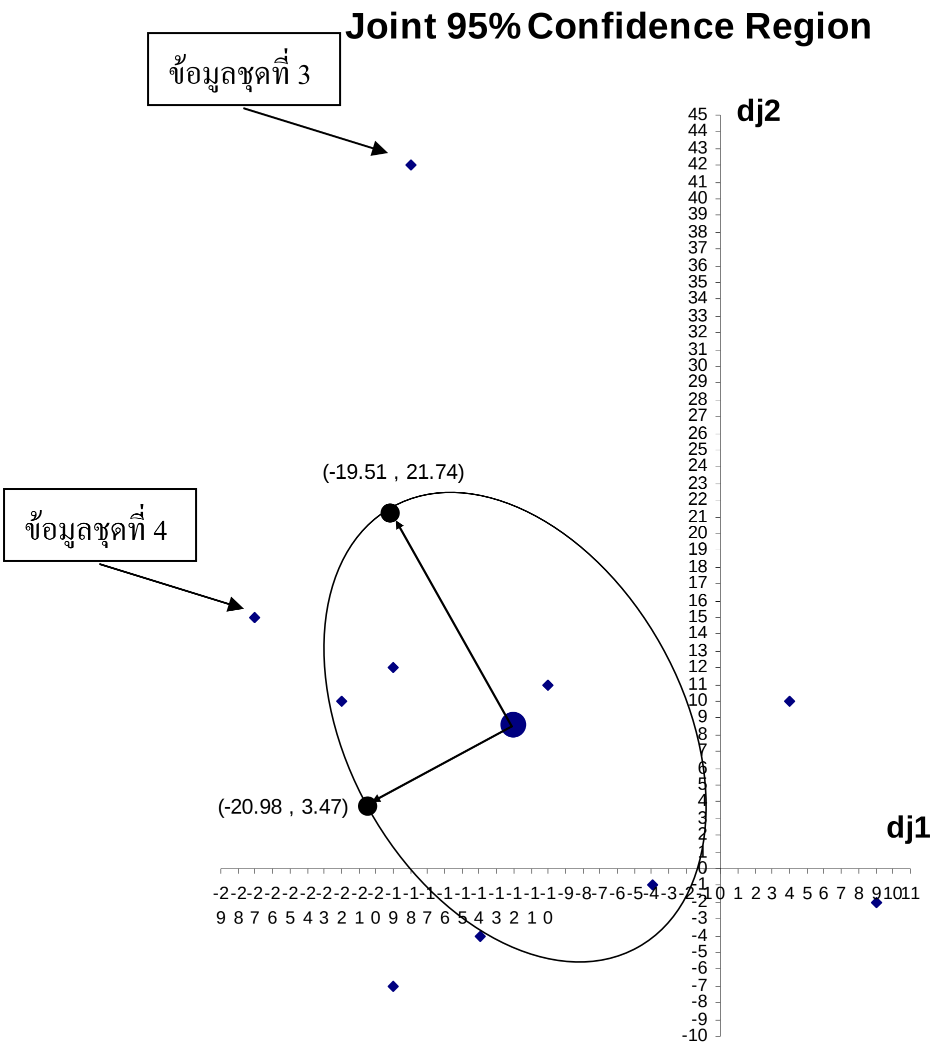
$$e_1 = \begin{bmatrix} -0.4961 \\ 0.8683 \end{bmatrix} \qquad e_2 = \begin{bmatrix} -0.8683 \\ -0.4961 \end{bmatrix}$$

$$\pm\sqrt{\lambda_1}\sqrt{\frac{p(n-1)}{n(n-p)}F_{p,n-p}(\alpha)} = \quad 15.10728 \quad x \quad 1.001748 \quad = \quad 15.1337$$

$$\pm\sqrt{\lambda_1}\sqrt{\frac{p(n-1)}{n(n-p)}F_{p,n-p}(\alpha)e_1} = \begin{bmatrix} -7.51 \\ 13.14 \end{bmatrix}$$

$$\pm\sqrt{\lambda_2}\sqrt{\frac{p(n-1)}{n(n-p)}F_{p,n-p}(\alpha)} = \quad 10.31891 \quad x \quad 1.001748 \quad = \quad 10.33696$$

$$\pm\sqrt{\lambda_2}\sqrt{\frac{p(n-1)}{n(n-p)}F_{p,n-p}(\alpha)e_2} = \begin{bmatrix} -8.98 \\ -5.13 \end{bmatrix}$$



รูปที่ 1 Joint 95% Confidence Region

The 95% Bonferroni simultaneous intervals for the components of the mean difference vector

$$\bar{x}_i - t_{n-1,(\alpha/2p)}\sqrt{\frac{s_{ii}}{n}} \le \delta_i \le \bar{x}_i + t_{n-1,(\alpha/2p)}\sqrt{\frac{s_{ii}}{n}}$$

$$\bar{x}_i - t_{9,(0.05/4)}\sqrt{\frac{s_{ii}}{9}} \le \delta_i \le \bar{x}_i + t_{9,(0.05/4)}\sqrt{\frac{s_{ii}}{9}}$$

$$\bar{x}_i - 2.685011\sqrt{\frac{s_{ii}}{9}} \le \delta_i \le \bar{x}_i + 2.685011\sqrt{\frac{s_{ii}}{9}}$$

$$-21.918 \le \delta_1 \le -2.082$$

$$-3.35559 \le \delta_2 \le 20.55559$$

Are the results consistent with a test of $H_o : \delta = 0$? Discuss.

Yes.

- จากการทดสอบ $H_o : \delta = 0$ แล้วในช่วงต้น

คือ $T^2 = 11.44652 > 3.167807$ We reject Ho and conclude that there is a nonzero mean difference between the measurements of the two laboratories.

เมื่อพิจารณา Bonferroni พบว่าผลการสร้างช่วงความเชื่อมั่นที่ 95% พบว่ามีค่าเฉลี่ยของ δ_1 ที่มีช่วงความเชื่อมั่น 95% ของ Bonferroni ไม่ครอบคลุม 0 แสดงว่า BOD ของ Commercial lab มีค่าเฉลี่ยแตกต่างจาก BOD ของ State lab of hygiene

Does the "outlier" make a difference in the analysis of these data?

Yes.

เมื่อพิจารณาว่ามี Outlier เกิดขึ้นหรือไม่ พบว่า **ข้อมูลชุดที่ 4** มีค่า Outlier จาก Confidence Region ทางด้านการวัด BOD มากที่สุด แต่ **ข้อมูลชุดที่ 3** ก็เป็นค่า Outlier สูงเช่นเดียวกัน แต่ค่านี้ส่งผลกระทบต่อการวัด SS ด้วย (ดูรูปที่ 1 ประกอบ)

6.5. A researcher considered three indices measuring the severity of heart attacks. The values of these indices for n = 40 heart-attack patients arriving at a hospital emergency room produced the summary statistics

$$\bar{x} = \begin{bmatrix} 46.1 \\ 57.3 \\ 50.4 \end{bmatrix} \quad \text{and } S = \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ 63.0 & 80.2 & 55.6 \\ 71.0 & 55.6 & 97.4 \end{bmatrix}$$

a) All three indices are evaluated for each patient. Test for the equality of mean indices using (6-16) with $\alpha = .05$

1. $H_o : \begin{bmatrix} \mu_1 - \mu_2 \\ \mu_1 - \mu_3 \end{bmatrix} = 0$ or $C\mu = 0$

   $H_1 : \begin{bmatrix} \mu_1 - \mu_2 \\ \mu_1 - \mu_3 \end{bmatrix} \neq 0$ or $C\mu \neq 0$

2. $\alpha = 0.05$

3. $T^2 = n(C\bar{x})'(CSC')^{-1}C\bar{x}$

$$C\bar{x} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 46.1 \\ 57.3 \\ 50.4 \end{bmatrix}$$

$$CSC' = \left(\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 101.3 & 63 & 71 \\ 63 & 80.2 & 55.6 \\ 71 & 55.6 & 97.4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \right)$$

$$= (40) \begin{bmatrix} -11.2 & -4.3 \end{bmatrix} \begin{bmatrix} 0.021621 & -0.00873 \\ -0.00873 & 0.021163 \end{bmatrix} \begin{bmatrix} -11.2 \\ -4.3 \end{bmatrix}$$

$$= (40) \begin{bmatrix} -0.2046 & 0.0068 \end{bmatrix} \begin{bmatrix} -11.2 \\ -4.3 \end{bmatrix}$$

$$= 90.49458$$

$T^2 = 90.49458$

4. $\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha) = \frac{(40-1)(3-1)}{(40-3+1)} 3.24 = 6.650526$

5. $T^2 = 90.49458 > \frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha) = 6.650526$

พวกเราจะ Reject $H_o : C\mu = 0$ ที่ระดับนัยสำคัญ 0.05 แสดงว่ามีการทำ Contrasts อย่างน้อยบางตัวทำให้ค่าเฉลี่ยคู่นั้นแตกต่างกัน

b) Judge the differences in pairs of mean indices using 95% simultaneous confidence intervals.

$$c'\mu : c'\overline{x} \pm \sqrt{\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha)} \sqrt{\frac{c'Sc}{n}}$$

ถ้าหา c_1

$$c_1'\mu : -11.2 \pm 2.579(1.178)$$

$$-14.2377 \le \mu_1 - \mu_2 \le -8.1623$$

ถ้าหา c_2

$$c_2'\mu : -4.3 \pm 2.579(1.191)$$

$$-7.3036 \le \mu_1 - \mu_3 \le -1.22964$$

แสดงว่า μ_1 มีค่าแตกต่าง(น้อยกว่า)จาก μ_2 และ μ_3

Note: การตรวจสอบนี้ไม่ได้ตรวจสอบทุกๆ Combination ของค่าเฉลี่ย เช่น Contrast ของ μ_2 และ μ_3 นั้นคือเป็นเพียงการหาช่วงความเชื่อมั่นจาก Contrasts ที่กำหนด C เท่านั้น

แต่ถ้าต้องการหา μ_2 - μ_3 สามารถจัดทำได้ดังนี้

$$c'\mu = [0 \quad 1 \quad -1] \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} : 6.9 \pm 2.579(1.288)$$

$$3.5774 \le \mu_2 - \mu_3 \le 10.2226$$

สามารถอธิบายเพิ่มเติมได้ว่า ทุกค่าเฉลี่ยมีค่าแตกต่างกัน

และการตรวจสอบการ Reject H_o ในข้อ a) ควรใช้ Bonferroni ตรวจสอบจะให้ผลลัพธ์ที่ดีกว่า (หนังสือหน้า 276 อธิบายเหตุผล)

6.7. Using the summary statistics for the electricity-demand data given in Example 6.4, compute T^2 and test the hypothesis $H_o : \mu_1 - \mu_2 = 0$, assuming that $\Sigma_1 = \Sigma_2$. Set $\alpha = .05$. Also, determine the linear combination of mean components most responsible for the rejection of H_o.

1. $H_o : \mu_1 - \mu_2 = 0$

   $H_o : \mu_1 - \mu_2 \neq 0$

2. $\alpha = .05$

3. $T^2 = (\bar{x}_1 - \bar{x}_2 - 0)' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S_{pooled} \right]^{-1} (\bar{x}_1 - \bar{x}_2 - 0)$

$$= \left(\begin{bmatrix} 204.4 \\ 556.6 \end{bmatrix} - \begin{bmatrix} 130.0 \\ 355.0 \end{bmatrix} \right)' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \begin{bmatrix} 10963.7 & 21505.5 \\ 21505.5 & 63661.3 \end{bmatrix} \right]^{-1} \left(\begin{bmatrix} 204.4 \\ 556.6 \end{bmatrix} - \begin{bmatrix} 130.0 \\ 355.0 \end{bmatrix} \right)$$

$T^2 = 16.066$

4. $c^2 = \frac{98(2)}{97} F_{2,97}(.05) = 6.26$

5. $T^2 = 16.066 > 6.26$

พวกเรา Reject $H_o : \mu_1 - \mu_2 = 0$ ที่ระดับนัยสำคัญ 0.05 แสดงว่ามีค่าเฉลี่ยอย่างน้อย 1 คู่แตกต่างกัน ดังนั้นจะใช้ Bonferrni 95% simultaneous confidence intervals เพื่อ p population mean differences เป็น

$$\mu_{1i} - \mu_{2i} : (\bar{x}_{1i} - \bar{x}_{2i}) \pm t_{n_1+n_2-2,(\alpha/2p)} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2} \right) s_{ii,pooled}}$$

หา $\mu_{11} - \mu_{21}$ $: (204.4 - 130.0) \pm 2.276362 \sqrt{\left(\frac{1}{45} + \frac{1}{55} \right) 10963.7}$

$$26.489 \leq \mu_{11} - \mu_{21} \leq 122.311$$

หา $\mu_{12} - \mu_{22}$ $: (556.6 - 355.0) \pm 2.276362 \sqrt{\left(\frac{1}{45} + \frac{1}{55} \right) 63661.3}$

$$86.151 \leq \mu_{11} - \mu_{21} \leq 317.049$$

We conclude that there is a difference in electrical consumption between those with air-conditioning and those without. This difference is evident in both onpeak and off-peak consumption

6.8. Observations on two responses are collected for three treatments. The observation vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are

Treatment 1: $\begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

Treatment 2: $\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Treatment 3: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

a) Break up the observations into mean, treatment, and residual components, as in (6-35). Construct the corresponding arrays for each variable.

$$\bar{x}_1 = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \bar{x}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \bar{x}_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \bar{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$x_{lj} = \bar{x} + (\bar{x}_l - \bar{x}) + (x_{lj} - \bar{x}_l)$$

$$\begin{pmatrix} 6 & 5 & 8 & 4 & 7 \\ 3 & 1 & 2 & & \\ 2 & 5 & 3 & 2 & \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & & \\ 4 & 4 & 4 & 4 & \end{pmatrix} + \begin{pmatrix} 2 & 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & & \\ -1 & -1 & -1 & -1 & \end{pmatrix} + \begin{pmatrix} 0 & -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & & \\ -1 & 2 & 0 & -1 & \end{pmatrix}$$

and

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$

$$246 = 192 + 36 + 18$$

Total SS (corrected) $= SS_{obs} - SS_{mean} = 246 - 192 = 54$

Repeating this operations on the second variable, we have

$$\begin{pmatrix} 7 & 9 & 6 & 9 & 9 \\ 3 & 6 & 3 & & \\ 3 & 1 & 1 & 3 & \end{pmatrix} = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & & \\ 5 & 5 & 5 & 5 & \end{pmatrix} + \begin{pmatrix} 3 & 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & & \\ -3 & -3 & -3 & -3 & \end{pmatrix} + \begin{pmatrix} -1 & 1 & -2 & 1 & 1 \\ -1 & 2 & -1 & & \\ 1 & -1 & -1 & 1 & \end{pmatrix}$$

and

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$

$$402 = 300 + 84 + 18$$

Total SS (corrected) $= SS_{obs} - SS_{mean} = 402 - 300 = 102$

Cross product

Mean : =240

Treatment: = 48

Residual: = -13

Total: = 275

Total (corrected) cross product = 275 – 240 = 35

b) Using the information in Part a, construct the one-way MANOVA table.

The MANOVA table

Source of variation	Matrix of sum of squares and cross products	Degrees of freedom
Treatment	$\begin{bmatrix} 36 & 48 \\ 48 & 84 \end{bmatrix}$	3-1 = 2
Residual	$\begin{bmatrix} 18 & -13 \\ -13 & 18 \end{bmatrix}$	5+3+4-3 = 9
Total (corrected)	$\begin{bmatrix} 54 & 35 \\ 35 & 102 \end{bmatrix}$	11

c) Evaluate Wilks' lambda. Λ^* ,and use Table 6.3 to test for treatment effects.Set $\alpha = .01$. Repeat the test using the chi-square approximation with Barlett's correction. [See (6-39).] Compare the conclusions.

Evaluate Wilks' lambda. Λ^* ,and use Table 6.3 to test for treatment effects.Set $\alpha = .01$.

1. $H_o : \tau_1 = \tau_2 = \tau_3 = 0$
2. $\alpha = .01$
3. $$\Lambda^* = \frac{|\mathrm{W}|}{|\mathrm{B+W}|} = \frac{\left|\begin{bmatrix} 18 & -13 \\ -13 & 18 \end{bmatrix}\right|}{\left|\begin{bmatrix} 54 & 35 \\ 35 & 102 \end{bmatrix}\right|} = \frac{155}{4283} = 0.0362$$

From Table 6.3 p=2 , g=3

$$\left(\frac{\sum n_l - g - 1}{g - 1}\right)\left(\frac{1-\sqrt{\Lambda *}}{\sqrt{\Lambda *}}\right) = 17.02$$

4. $F_{2(g-1),2(\sum n_l-g-1)}(0.01) = F_{4,16}(0.01) = 4.77$

5. 17.02 > 4.77 พวกเราจะ Reject $H_o:\tau_1=\tau_2=\tau_3=0$ **ที่** $\alpha=.01$ **แสดงว่ามีความแตกต่างระหว่าง Treatment เกิดขึ้น**

Repeat the test using the chi-square approximation with Barlett's correction. [See (6-39).] Compare the conclusions.

เริ่มขั้นตอนที่ 3. เพื่อเปรียบเทียบตัวสถิติทดสอบทั้ง 2 แบบ

3. $$\Lambda^* = \frac{|\mathrm{W}|}{|\mathrm{B+W}|} = \frac{\left|\begin{bmatrix}18 & -13\\ -13 & 18\end{bmatrix}\right|}{\left|\begin{bmatrix}54 & 35\\ 35 & 102\end{bmatrix}\right|} = \frac{155}{4283} = 0.0362$$

$$-\left(n-1-\frac{(p+g)}{2}\right)\ln\left(\frac{|\mathrm{W}|}{|\mathrm{B+W}|}\right) = -\left(12-1-\frac{(2+3)}{2}\right)\ln\left(0.0362\right)$$

$= 28.209$

4. $\chi^2_{p(g-1)}(\alpha) = \chi^2_4(0.01) = 13.28$

5. 28.209 > 13.28 พวกเราจะ Reject $H_o:\tau_1=\tau_2=\tau_3=0$ **ที่** $\alpha=.01$ **แสดงว่ามีความแตกต่างระหว่าง Treatment เกิดขึ้น ซึ่งผลสรุปนี้เหมือนกันกับการใช้ Table 6.3 ที่เปิดตาราง F ทดสอบ แสดงว่าการใช้ chi-square approximation with Barlett's correction ได้ผลการทดสอบสมมติฐานพอใช้ประมาณได้ ซึ่งจะได้ผลการประมาณดีขึ้นถ้า n มีขนาดใหญ่**

6.13. (Two-way MANOVA without replications.) Consider the observations on two responses, x_1 and x_2, displayed in the form of the following two-way table(note that there is a single observation vector at each combination of factor levels):

		Factor 2			
		Level 1	Level 2	Level 3	Level 4
Factor 1	Level 1	$\begin{bmatrix}6\\8\end{bmatrix}$	$\begin{bmatrix}4\\6\end{bmatrix}$	$\begin{bmatrix}8\\12\end{bmatrix}$	$\begin{bmatrix}2\\6\end{bmatrix}$
	Level 2	$\begin{bmatrix}3\\8\end{bmatrix}$	$\begin{bmatrix}-3\\2\end{bmatrix}$	$\begin{bmatrix}4\\3\end{bmatrix}$	$\begin{bmatrix}-4\\3\end{bmatrix}$
	Level 3	$\begin{bmatrix}-3\\2\end{bmatrix}$	$\begin{bmatrix}-4\\-5\end{bmatrix}$	$\begin{bmatrix}3\\-3\end{bmatrix}$	$\begin{bmatrix}-4\\-6\end{bmatrix}$

With no replications, the two-way MANOVA model is

$$X_{lk} = \mu + \tau_l + \beta_k + e_{lk}; \quad \sum_{l=1}^{g} \tau_l = \sum_{k=1}^{b} \beta_k = 0$$

Where the e_{lk} are independent $N_p(0, \Sigma)$ random vectors.

a) Decompose the observations for each of the two variables as

$$x_{lk} = \bar{x} + (\bar{x}_{l.} - \bar{x}) + (\bar{x}_{.k} - \bar{x}) + (x_{lk} - \bar{x}_{l.} - \bar{x}_{.k} + \bar{x})$$

Similar to the arrays in Example 6.8. For each response, this decomposition will result in several 3 x 4 matrices. Here $\bar{x}$ is the overall average, $\bar{x}_{l.}$ is the average for the lth level of factor 1, and $\bar{x}_{.k}$ is the average for the kth level of factor 2.

Factor 1

$$\bar{x}_{1.} = \begin{bmatrix}5\\8\end{bmatrix}, \bar{x}_{2.} = \begin{bmatrix}0\\4\end{bmatrix}, \bar{x}_{3.} = \begin{bmatrix}-2\\-3\end{bmatrix}$$

Factor 2

$$\bar{x}_{.1} = \begin{bmatrix}2\\6\end{bmatrix}, \bar{x}_{.2} = \begin{bmatrix}-1\\1\end{bmatrix}, \bar{x}_{.3} = \begin{bmatrix}5\\4\end{bmatrix}, \bar{x}_{.4} = \begin{bmatrix}-2\\1\end{bmatrix}$$

$$\bar{x} = \begin{bmatrix}1\\3\end{bmatrix}$$

$$x_{lk} = \bar{x} + (\bar{x}_{l.} - \bar{x}) + (\bar{x}_{.k} - \bar{x}) + (x_{lk} - \bar{x}_{l.} - \bar{x}_{.k} + \bar{x})$$

Response x_1

$$\begin{pmatrix} 6 & 4 & 8 & 2 \\ 3 & -3 & 4 & -4 \\ -3 & -4 & 3 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 4 & 4 \\ -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 4 & -3 \\ 1 & -2 & 4 & -3 \\ 1 & -2 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 1 & -1 & 0 \\ 2 & -1 & 0 & -1 \\ -2 & 0 & 1 & 1 \end{pmatrix}$$

and response x_2

$$\begin{pmatrix} 8 & 6 & 12 & 6 \\ 8 & 2 & 3 & 3 \\ 2 & -5 & -3 & -6 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{pmatrix} + \begin{pmatrix} 5 & 5 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ -6 & -6 & -6 & -6 \end{pmatrix} + \begin{pmatrix} 3 & -2 & 1 & -2 \\ 3 & -2 & 1 & -2 \\ 3 & -2 & 1 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 0 & 3 & 0 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & -1 & -1 \end{pmatrix}$$

b) Regard the rows of the matrices in Part a as strung out in a single "long" vector, and compute the sums of squares

$SS_{tot} = SS_{mean} + SS_{fac1} + SS_{fac2} + SS_{res}$

And sums of cross products

$SCP_{tot} = SCP_{mean} + SCP_{fac1} + SCP_{fac2} + SCP_{res}$

Consequently, obtain the matrices $SSP_{cor}, SSP_{fac1}, SSP_{fac2}$, and SSP_{res} with degree of freedom gb-1, g-1, b-1, and (g-1)(b-1), respectively.

Response x_1

$SS_{tot} = SS_{mean} + SS_{fac1} + SS_{fac2} + SS_{res}$

$220 = 12 + 104 + 90 + 14$

Total SS (corrected) $= SS_{tot} - SS_{mean} = 220 - 12 = 208$

and response x_2

$SS_{tot} = SS_{mean} + SS_{fac1} + SS_{fac2} + SS_{res}$

$440 = 108 + 248 + 54 + 30$

Total SS (corrected) $= SS_{tot} - SS_{mean} = 440 - 108 = 332$

Cross Products

$SCP_{tot} = SCP_{mean} + SCP_{fac1} + SCP_{fac2} + SCP_{res}$

$227 = 36 + 148 + 51 - 8$

Total (corrected) Cross Product $= SCP_{tot} - SCP_{mean} = 227 - 36 = 191$

$SSP_{cor} = \begin{bmatrix} 208 & 191 \\ 191 & 332 \end{bmatrix}$,degree of freedom gb-1 = 11

$SSP_{fac1} = \begin{bmatrix} 104 & 148 \\ 148 & 248 \end{bmatrix}$, degree of freedom g-1 = 2

$SSP_{fac2} = \begin{bmatrix} 90 & 51 \\ 51 & 54 \end{bmatrix}$, degree of freedom b-1 = 3

and $SSP_{res} = \begin{bmatrix} 14 & -8 \\ -8 & 30 \end{bmatrix}$,degree of freedom (g-1)(b-1) = 6

c) Summarize the calculations in Part b in a MANOVA table.

The MANOVA table

Source of variation	Matrix of sum of squares and cross products	Degrees of freedom
Factor 1	$\begin{bmatrix} 104 & 148 \\ 148 & 248 \end{bmatrix}$	2
Factor 2	$\begin{bmatrix} 90 & 51 \\ 51 & 54 \end{bmatrix}$	3
Residual (Error)	$\begin{bmatrix} 14 & -8 \\ -8 & 30 \end{bmatrix}$	6
Total (corrected)	$\begin{bmatrix} 208 & 191 \\ 191 & 332 \end{bmatrix}$	11

d) Given the summary in Part c,test for factor 1 and factor 2 main effects at the $\alpha = .05$ level.

Factor 1

1. $H_o : \tau_1 = \tau_2 = \tau_3 = 0$
2. $\alpha = .05$
3. $\Lambda^* = \dfrac{|SSP_{res}|}{|SSP_{fac1} + SSP_{res}|} = \dfrac{356}{13204} = 0.026962$

$$-\left[(g-1)(b-1) - \frac{p+1-(g-1)}{2}\right]\ln \Lambda^* = -\left[6 - \frac{2+1-(3-1)}{2}\right]\ln 0.026962$$

$= 19.8733$

4. $\chi^2_{(g-1)p}(\alpha) = \chi^2_4(0.05) = 9.49$
5. 19.8733 > 9.49 **พวกเราจะ Reject** $H_o : \tau_1 = \tau_2 = \tau_3 = 0$ **ที่** $\alpha = .05$ **แสดงว่าการเปลี่ยน Level ของ Factor 1 มีผลต่อ Reponses** x_1 **(และ)หรือ** x_2 **(มีผลอย่างไรต้องทำการตรวจสอบต่อไป: Simultaneous confidence intervals for contrasts in the components of** τ_l **are considered.)**

Factor 2

1. $H_o : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$
2. $\alpha = .05$
3. $\Lambda^* = \dfrac{|SSP_{res}|}{|SSP_{fac1} + SSP_{res}|} = \dfrac{356}{6887} = 0.051692$

$$-\left[(g-1)(b-1) - \frac{p+1-(b-1)}{2}\right]\ln \Lambda^* = -\left[6 - \frac{2+1-(4-1)}{2}\right]\ln 0.051692$$

$= 17.7747$

4. $\chi^2_{(b-1)p}(\alpha) = \chi^2_6(0.05) = 12.59$
5. 17.7747 > 12.59 **พวกเราจะ Reject** $H_o : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ **ที่** $\alpha = .05$ **แสดงว่าการเปลี่ยน Level ของ Factor 2 มีผลต่อ Reponses** x_1 **(และ)หรือ** x_2 **(มีผลอย่างไรต้องทำการตรวจสอบต่อไป Simultaneous confidence intervals for contrasts in the components of** β_k **are considered.)**