THE DYNAMICS OF PLANT LAYOUT*

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The problem of plant layout has generally been treated as a static one. In this paper, we deal with the dynamic nature of this problem. Both optimal and heuristic procedures are developed for this problem and are based on a dynamic programming formulation. The use of one of these approaches depends on the ability to solve the static problem efficiently. Finally, we briefly discuss the issue of extending the planning horizon, and how to resolve system nervousness when previously planned layouts need to be changed.

(FACILITIES/EQUIPMENT PLANNING—LAYOUT; DYNAMIC PROGRAMMING—APPLICATIONS; NETWORKS/GRAPHS)

Introduction

It is estimated that about eight percent of the U.S. gross national product has been annually spent since 1955 on new facilities. In addition, a significant percentage of previously purchased facilities are modified. These data imply that over $250 billion is annually spent in the U.S. alone on facilities that require planning or replanning (see Tompkins and White 1984). The importance of the subject of plant layout and material handling is further suggested by Tompkins and White, who claim that: “It had been estimated that between 20% to 50% of the total operating expenses within manufacturing are attributed to material handling. Effective facilities planning can reduce these costs by at least 10% to 30% and thus increase productivity,” (Ibid, p. 5). This claim is supported by a recent survey among 33 companies in Great Britain (Nicol and Hollier 1983). Nicol and Hollier observed that the labor costs of personnel employed in handling, storage and transport duties are about 12% of total work labor costs.

The problem of plant, or facilities, layout has been generally treated as a static one. In this paper, we intend to deal with the dynamic nature of this problem. The need of a dynamic treatment of the plant layout problem is supported by Nicol and Hollier (1983), who concluded in their study that: “Radical layout changes occur frequently and that management should therefore take this into account in their forward planning.” Furthermore, if the effective lifetime of a layout is defined as the elapsed time from installation until at least one-third of all key manufacturing operations are replaced, then it was found that nearly half of the companies surveyed had an average layout stability of two years or less. The mean of all firms was just over three years and was shorter for the engineering companies (Nicol and Hollier 1983).

The plant layout problem is concerned with an arrangement of physical facilities (departments, machines). Two objective functions (quantitative and qualitative) are usually being optimized.

A quantitative objective is that of minimizing the material handling cost, and a qualitative objective is that of maximizing some measure of closeness ratings. Heuristic procedures (Armour and Buffa 1963, Drezner 1980, Vollmann, Nugent and Zartler 1968, Hillier 1963 and Hillier and Connors 1966) as well as optimal procedures for small size problems (Gilmore 1962, Lawler 1963) have been developed for minimizing

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the material handling costs. Computerized packages that solve the plant layout problem are available, to mention a few, CRAFT (Buffa, Armour and Vollmann 1964), COFAD (Tompkins and Reed 1973, 1976), ALDEP (Seehof and Evans 1967), and CORELAP (Lee and Moore 1967, Moore 1971). The last two computerized approaches are used to maximize some measure of closeness rating, whereas the first two minimize the total material flow (distance) cost. A heuristic algorithm has been developed for combining these two quantitative and qualitative approaches. This algorithm results in an efficient frontier set which includes only “efficient layouts” (Rosenblatt 1979). Also, some three-dimensional plant layout packages were developed, see CRAFT-3D (Cinar 1975), SPACECRAFT (Johnson 1982), and for a brief comparison between those two, see Jacobs (1984).

In the following section, the static plant layout problem is presented. Then it is extended to a multi-period model. A dynamic programming model is developed and solution procedures, both optimal and heuristic, are suggested. Finally, the issue of extending the planning horizon (rolling schedule) is discussed.

The Static Plant Layout Problem (SPLP)

The Static Plant Layout Problem (SPLP) minimizes the total material handling costs associated with assigning the different facilities to the various locations and is usually formulated as a quadratic assignment problem. In this formulation, the number of locations is equal to the number of departments (facilities). However, as is shown in Hillier and Connors (1966), every problem can be modified to this structure by introducing dummy departments or locations into the problem.

The SPLP Model

\[
\text{Min } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ijkl} x_{ij} x_{kl} \quad \text{s.t.} \quad (1)
\]

\[
\sum_{j=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, n \quad (2)
\]

\[
\sum_{i=1}^{n} x_{ij} = 1, \quad i = 1, \ldots, n, \quad (3)
\]

\[
x_{ij} = 0 \text{ or } 1, \quad \text{where:} \quad (4)
\]

\[
x_{ij} = \begin{cases} 
1 & \text{if department } i \text{ is assigned to location } j, \\
0 & \text{otherwise.} 
\end{cases} \quad (5)
\]

\[
a_{ijkl} = \begin{cases} 
 f_{ik} d_{ij} & \text{if } i \neq k \text{ or } j \neq l, \\
 f_{i} d_{ij} + c_{ij} & \text{if } i = k \text{ and } j = l, \text{ and} 
\end{cases} \quad (6)
\]

\(c_{ij}\) = cost per unit time associated directly with assigning department \(i\) to location \(j\),
\(d_{ij}\) = “distance” from location \(j\) to location \(l\) (travel cost between locations); where \(d_{ii} = 0\).
\(f_{ik}\) = work flow from department \(i\) to department \(k\).

As was mentioned earlier, different solution procedures, both heuristic and optimal, were suggested for solving this quadratic assignment problem. However, as is noted by Francis and White (1974), “except for relatively small-sized problems, an exact solution to the quadratic assignment problem cannot be obtained at a reasonable computational cost. Therefore, heuristic solution procedures are generally used to obtain ‘good’ solutions to the quadratic assignment problem.” Ritzman (1972) compared different heuristic algorithms for this problem and concluded that the CRAFT
algorithm is probably the best, followed by the Hillier-Connors procedure. Interesting
sets of experiments were also conducted comparing Human vs. Computer algorithm
performance to the plant layout problem. As should be expected, the computer
algorithm performs better as the problem size becomes larger and/or with a low flow
dominance. These results are summarized in Trybus and Hopkins (1980).

A common element of all of the above procedures is that they solve a static plant
layout problem. Based on the current state of the business (current flow pattern), these
procedures result in a plant layout configuration. Thus, for example, in a job shop
environment, a procedure like CRAFT, which is based on the flow data between the
different facilities, will result in a layout which is a function of current production
orders and technology. However, given the dynamic nature of the business (new
manufacturing orders, new product lines, technological advances), some changes in the
current layout may be desirable in the future. In the following section, a dynamic
programming formulation is presented for the long-run plant layout problem.

The Dynamic Plant Layout Problem (DPLP)

A deterministic environment is assumed, where the number of orders and the
quantities, arrival and due dates for the different products (or family of products) are
known for a given finite horizon. For convenience, this data is summarized in a
"From-To" flow matrix for each of the coming periods. Depending on the nature of
the business, a period can be given in terms of months, quarters, years, etc. The major
question involved in the Dynamic Plant Layout Problem (DPLP) is what should be the
layout in each period, or to what extent, if any, should changes in the layout be made.

The costs associated with the DPLP are those pertaining to material flow and those
involved with rearrangements of the layouts. The material flow costs are introduced in
the SPLP model and are a product of flow and distance. For simplicity it will be
assumed that the initial cost of assigning department \( i \) to any location \( j \) is independent
of the location. Thus, the term \( c_{ij} \) in (6) may be ignored. However, rearranging the
layout will result in some shifting costs depending on the departments involved in this
shift. The rearrangement (shifting) costs may be viewed as fixed costs, or costs
depending on the departments involved in the change, or costs depending on the
departments involved and the distance between the various locations, or any combina-
tion of the above. For our analysis, it is assumed that there is a cost vector which
represents the cost involved in moving a specific department from its location. These
costs may be modified to incorporate the distance between the two departments
involved, see Driscoll and Sawyer (1983).

The maximum number of different layouts in any period is \( n! \), where \( n \) is the
number of departments or locations. If symmetry is considered, then this number can
be divided by the symmetry measure (e.g., a symmetry measure of a quadrangle is 4).
However, in the dynamic version of the problem, all different layouts (even if
symmetrical) must be considered, since different shifting costs will result from the
different, although symmetrical, layouts. Furthermore, assuming \( T \) periods are consid-
ered, then the maximum number of combinations that needs to be considered is \( (n!)^T \).
It is clear that for any reasonable value of \( n \), a total enumeration procedure may be
computationally prohibitive. Therefore, a dynamic programming approach is sug-
gested for this DPLP. Using dynamic programming terminology, a stage will corre-
spond to a period and a state will correspond to a specific layout. However, even in
dynamic programming, considering the total number of layout combinations in each
period may result in a very large problem, and thus a simplifying procedure is
warranted.

Let \( Z_{ir} \) denote the value of \( r \)th best solution to the SPLP for period \( t \). Then
$Z^{\text{inf}} = \sum_{i=1}^{T} Z_{i1}$ is the sum of the minimum costs of the SPLP in each period, for the entire planning horizon. Since no rearrangement costs are considered, it is clear that $Z^{\text{inf}}$ is a lower bound on the value of the optimal multi-period solution. "Time value of money" (discounting) considerations are excluded from this analysis, but can easily be incorporated. Let $Z^{UB}$ be an upper bound corresponding to the best incumbent feasible solution to the multi-period problem (rearrangement costs included). Then, using the Sweeney and Tatham (1976) theorem for the long-run multiple warehouse location problem, the following claim can be stated.

**Claim.** If $K = Z^{UB} - Z^{\text{inf}}$ and $R_i$ is given by $Z_{i,R_i} - Z_{i1} \leq K$ and $Z_{i,R_i+1} - Z_{i1} > K$, then, in any period $i$, no static solution with value $r > R_i$ may become part of an optimal solution.

The proof of this claim is straightforward and is found in Sweeney and Tatham (1976). The implication of this claim is that there is no need theoretically to evaluate all the different $n!$ possible layouts for each period; possibly a smaller number could provide the optimal solution. Two problems still remain to be answered. The first is how to get a good (small) upper bound on the value of $Z$, and the second involves getting the $R_i$ best ranked solutions to the SPLP.

Several procedures may be used for generating an upper bound value, $Z^{UB}$. One possibility is to continue with the current optimal layout of the first period for all the periods. A modification of this approach is to select the same layout for the entire horizon period. Candidates for this layout are obtained from the set of best solutions (layouts) in each one of the periods. The layout which results in the lowest cost is used for getting an upper bound value. In this approach, at most $T$ different possibilities need to be compared. Another approach is to consider the best layout for each period. In this case, the upper bound on the total cost is $Z^{\text{inf}}$ plus costs involved in rearrangements. The minimum between those two possibilities will depend on the nature of the problem, more specifically, on the relationships between the variation of the flow in the various periods and the rearrangement costs relative to the material handling costs. If rearrangement costs are relatively small, the second possibility will probably render a lower upper bound $Z^{UB}$. Obviously, this upper bound may be revised and updated during the dynamic programming solution procedure, as will be further discussed.

In order to find the best $R_i$ solutions for the SPLP model, the following procedure is suggested. After getting $Z_{i1}$, obtained by solving the SPLP model equations (1)-(6), an additional constraint is added to the SPLP model which precludes the current solution. Solving the original SPLP with the additional constraint results in $Z_{i2}$. Based on the solution of $Z_{i2}$, another constraint is added, resulting in $Z_{i3}$, and so on. The general form of the constraint to be added is:

$$\sum_{(i,j) \in A_k} X_{ij} - \sum_{(i,j) \in B_k} X_{ij} \leq n - 2$$

where

$$A_k = \{(i,j) : x_{ij} = 1\}, \quad B_k = \{(i,j) : x_{ij} = 0\}.$$ 

Thus, $A_1$ is the set of $x_{ij}$'s equal to 1 in the best solution for the SPLP for a given period.

**Formulation and Solution Procedures**

Let

- $S_t$ = Set of all layouts to be considered in period $t$.
- $C_{km}$ = Rearrangement (shifting) costs from layout $A_k$ to layout $A_m$, where $C_{A_kA_k} = 0$.
- $Z_{ik}$ = Material handling costs for layout $A_k$ in period $t$. This can be obtained from the SPLP solution.
\( L_{it}^* = \) Minimum total costs (material handling and shifting) for all periods up to \( t \), where layout \( A_k \) is being used in period \( t \).

A layout is characterized by the set \( A_k \), which includes all \( x_{ij} = 1 \). Obviously, the rearrangement costs are zero if no change in the layout is made from one period to another. Furthermore, it is assumed that shifting costs are independent of the periods in which they are being made. However, this assumption can be easily relaxed, and \( C_{km} \) can be made a function of the period in which the change takes place. A recursive formulation is developed, and the total cost for each of the layouts considered in the horizon is obtained. Then, the combination of layouts with the minimum total cost is chosen. The following recursive relationship is established:

\[
L_{it}^* = \min_k \{ L_{i-1,k}^* + C_{km} \} + Z_{im}^*, \quad t = 1, \ldots, n,
\]

and \( L_{01}^* = 0 \), assuming there is a single initial layout.

This recursive relationship may be used to generate either an optimal or a heuristic solution. Obviously, if all the possible layouts are considered for each period, then an optimal solution is obtained by using this recursive relationship. However, computational time usually increases exponentially with the number of states in a dynamic programming problem. Therefore, the number of states to be considered should be reduced as much as possible. For a global optimal solution, only the best \( R \), layouts for each period need to be considered. If it is computationally feasible to solve the SPLP as many times as required, then this approach should be adopted. However, except for relatively small sized problems, 10 to 15 departments, solving the SPLP problem in an exact manner (e.g., Gilmore 1962, Lawler 1963) will be computationally prohibitive, let alone resolving it as many times as needed to get all the required \( R \)'s. Therefore, for those more general cases, a heuristic procedure is advocated. In this case, two approaches may be considered. The first one involves solving optimally the SPLP model for all periods. Then, the set of layouts to be considered in each period includes only the best layouts in all periods. Thus, the maximum number of layouts (states) in each period (stage) is \( n \). In the case where the best layout in different periods is the same, then the number of states is decreased accordingly. This heuristic procedure is similar to Ballou's (1968) for the warehouse location problem.

The second approach should apply when solving the SPLP optimally is computationally prohibitive. In this case, effective computerized approaches such as CRAFT or COFAD could be used to generate sets of solutions (layouts) for the different periods. Also, randomly generating different layouts may be considered. The number of layouts to be generated in each period will depend on the level of accuracy required by the solution. One way to evaluate the possibility of improving the solution is by comparing the incumbent upper and lower bounds of the global solution. In the following example we compare the optimal solution with the solutions obtained by randomly generating layouts in the various periods.

### Numerical Example

Consider a plant with six departments where, for simplicity (and without loss of generality), the departments are assumed to be of equal size. The horizon planning is five periods. The flow data between the different departments for the various periods are given by the following set of "From-To" matrices, \( F_i \), where \( F_i \) is the material handling flow for period \( i \). Also, a row vector \( z_i \) will represent the costs associated with shifting department \( i \) to another location, see Table 1. Again for the sake of simplicity, it is assumed that shifting costs are independent of the distances between the "old" and "new" locations and independent of the period in which the shift takes place.
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<th>From</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</table>

$S = (887 \ 964 \ 213 \ 367 \ 289 \ 477)$
Furthermore, we assume that the assignment costs of a department to a location, \( c_{ij} \), can be ignored. Given this data, the problem is to find the optimal layout for each one of the periods. A simple rectangular configuration of the six locations (departments) is assumed, as illustrated in Figure 1.

The lower bound on the optimal solution, \( Z^m \), is 66,698. This corresponds to the following (optimal static solutions) layouts in each one of the periods:

<table>
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<tr>
<th>Period</th>
<th>Layout</th>
<th>( Z_{11} )</th>
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<td>(1, 3, 5, 6, 4, 2)</td>
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<td>5</td>
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<td>12,819</td>
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</table>

An upper bound on the optimal solution can be obtained by adding the shifting costs to \( Z^m \). This corresponds to a myopic approach, in which the optimal layout is used in each period and rearrangement (shifting) costs are incurred. The total cost in this case is: 75,384. Another upper bound can be obtained by taking the minimum of the total costs, provided the same layout is used for the entire five periods. The following table summarizes these results. Note that the smallest upper bound is obtained if the best layout in the second period is used throughout the planning horizon. In this case, the smallest upper bound cost is 74,137, as illustrated in Table 2.

Considering the four best layouts in each period (these are symmetrical layouts) as possible states (layouts) in the dynamic programming approach results in a problem of five stages (periods) and 20 states (layouts) in each period. Total cost obtained using this heuristic approach (Ballou 1968) was 72,525 with the following layouts: (6, 4, 2, 1, 3, 5) in periods 1 and 2, (6, 4, 2, 3, 5, 1) in period 3, and (3, 5, 2, 4, 6, 1) and (3, 2, 6, 4, 1, 5) in periods 4 and 5, respectively. The optimal solution, however, required the following layouts as presented in Table 3.

The optimal total cost for this problem is 71,187. This optimal solution is essentially a savings of 4,197 (5.9%), compared to the myopic approach of selecting the best layout in each period, and a savings of 5,423 (7.6%) if the best layout for the first period is used throughout the five periods.

As mentioned in the previous section, we compared the optimal solution with randomly generated solutions. In the first experiment we generated ten different

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>Total Cost When the Best Layout in Period t Is Used throughout the Horizon Plan</th>
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<tbody>
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<td>Layout</td>
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<tr>
<td>(1, 5, 3, 2, 4, 6)</td>
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</tr>
<tr>
<td>(1, 6, 4, 2, 5, 3)</td>
<td>673</td>
</tr>
<tr>
<td>(3, 2, 6, 4, 1, 5)</td>
<td>629</td>
</tr>
</tbody>
</table>
layouts in each period and then solved the dynamic programming problem considering 50 states in each period. The total cost function using this approach is 72,228 (about 1.5% above the optimal solution) with the following layout (2, 4, 6, 1, 3, 5) for periods 1 and 2 and layout (2, 4, 6, 1, 5, 3) for periods 3–5. When 20 different layouts were generated for each period, for a total of 100 states (layouts) in each stage, the total cost became 71,984 (about 1.1% above the optimal solution) with the following layout (6, 3, 1, 5, 4, 2) for periods 1 and 2, and layout (6, 5, 1, 3, 4, 2) for periods 3–5. Obviously, one should be very careful in trying to generalize these results to other situations.

To measure the effectiveness of the Ballou method and the randomly generated layouts approach, the following experiments were conducted. Thirty different settings (different flow matrices and shifting cost vectors) were constructed. For each one of those settings, the two procedures were compared with the optimal solution, and the relative errors (in percentages) were recorded. In the Ballou method, only 20 layouts (the best four of each of the five periods) were considered in each stage (period) of the dynamic programming formulation. Whereas, in the randomly generated layout approach, 100 layouts (states), out of a possibility of 720 layouts, were considered in each period. These 100 layouts correspond to 20 layouts generated for each one of the five periods. The average errors reported for the Ballou and the randomly generated layout approach are 1.20% and 1.63%, respectively. The relative errors for both procedures are quite small; however one should realize that in the Ballou method the best four layouts in each period must be found, before the dynamic programming procedure can be applied.

An interesting feature of this example, and one found commonly in many other examples solved by the author, relates to the number of states that can be eliminated from the dynamic programming solution procedure. Even if $K$, the maximum value that the solution can still be improved, is calculated as the difference between the optimal solution and $Z^{inf}$, resulting in $K = 4,489$, no layout (state) can be eliminated from the dynamic programming solution procedure. (The difference between $Z_{UB}$ and $Z_{1t}$ for all $t$ in this example is less than $K$.) Obviously, if $K$ had been calculated as the difference between $Z^{UB}$ and $Z^{inf}$, then no layout could have been eliminated as a potential state in the dynamic programming solution. The implication of this is quite simple. It means that 720 layouts are needed to be considered in each stage in this example, if an optimal solution is required.

### Extending the Planning Horizon

In the previous sections it was assumed that flow data are known for a given horizon time. These data are based on actual orders that are currently known and that will extend into, or begin in, the future. However, in a dynamic environment, things may change and orders may either be cancelled or added. Assuming that data are usually available for $n$ periods, then after the first period, additional data for the coming $n$
periods may become available. Based on these new data, the "From-To" matrices are updated, and different layouts may be recommended for periods 2, 3, ..., n, n + 1.

Two cases need to be considered. The first is a simple case, in which no damage, financially or otherwise, is caused to the system by changing the layout plans for the next n periods. In this case, once new data are available, they should be incorporated and the new solution be implemented.

However, a more interesting, and perhaps more realistic, case is when there is some "changing costs" or "nervousness" costs incurred whenever a change in the layout planning is recommended based on the updated data. The treatment of this case can be done in a similar way to that suggested by Carlson et al. (1979) and Kropp and Carlson (1984), dealing with nervousness of MRP systems. There, system nervousness occurs when changes in the production schedule happens. Nervousness is mainly due to adding or cancelling setups in the various periods. The idea behind their algorithm is to "increase" the "effective" setup costs in those periods for which no setup was previously scheduled, and decrease the "effective" setup cost for periods having a scheduled setup, (see Kropp and Carlson, p. 244). A similar procedure may be applied for the dynamic plant layout problem.

Two types of nervousness in layouts may occur. One is when a change in layout takes place in a period where it has not been scheduled before, or when a previously suggested shift in layout in a given period is cancelled. Another type of nervousness is when a different change in layout is desired for a period for which initially a change in layout was scheduled. It is assumed here that the "changing costs" for the second type could usually be ignored; however, they may be significant for the first type. The reason is that usually the "changing" cost incurred by revising one plan to another is affected by previously made commitments in periods for which production was either scheduled or postponed. This is also consistent with the Carlson et al. (1979) treatment, where it is assumed that nervousness is due to changes in the setup schedule and not in the amount produced. Furthermore, given our definition of nervousness, a similar treatment, of incorporating "changing cost" into the dynamic programming formulation can be applied.

Conclusion

In a recent study by Rushton and Williams (1982, 1983), it was found that the total material handling cost element is equivalent to 39% of the total gross domestic product of Great Britain. Similar results are given for U.S. and Denmark where the proportion of material handling costs to total production costs is about 30% and 45%, respectively. Since these percentages are quite significant, reduction in material handling costs should be viewed as an important objective of the firm.

In this study we are dealing with one facet of the material handling cost in the context of plant layout. Whereas previously the plant layout problem has been treated as a static problem, we provide here a solution for the dynamic plant layout problem. The dynamic programming formulation presented can be used to solve the dynamic plant layout problem in an optimal or a heuristic manner. This depends on the computational efficiency with which the static (single period) problem can be solved. Since solving a problem with a large number of departments may be computationally prohibitive, heuristic approaches are recommended. Heuristic solutions are either a result of solving the SPLP heuristically (with any of the many algorithms available), or not considering all the possible states (layouts) in the dynamic programming formulation, or a combination of the two. As was experienced by the author, the number of states that can be eliminated from the dynamic programming procedure, if an optimal solution is desirable is not very large. Thus, for large problems it is recommended that
this dynamic programming solution procedure be used, but smaller number of states be considered, resulting in an effective heuristic solution.

In a job shop environment with frequent fluctuations in orders, the question of layout “nervousness” needs to be addressed. As a deterministic environment is assumed here, the “nervousness” of the plant layout planning and its effects on the planning horizon should be included in the dynamic programming analysis. This should be implemented in a similar framework to that applied to “nervousness” of MRP systems. Finally, one of the suggested avenues for future research is incorporating the stochastic nature of demand into the analysis of the dynamic plant layout problems.¹

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References


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