

HW1

“15” Fifteen Puzzle (Game by Sam Loyd)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The "15 puzzle" is a sliding square puzzle introduced by Sam Loyd in 1878. It consists of 15 squares numbered from 1 to 15 that are placed in a 4×4 box leaving one position out of the 16 empty. The goal is to reposition the squares from a given arbitrary starting arrangement by sliding them one at a time into the configuration shown above. For some initial arrangements, **this rearrangement is possible, but for others, it is not.**

To address the solubility of a given initial arrangement, proceed as follows. If the square containing the number i appears "before" (reading the squares in the box from left to right and top to bottom) n_i numbers that are less than i , then call it an inversion of order n_i , and denote it n_i .

Then define

$$N \equiv \sum_{i=1}^{15} n_i = \sum_{i=2}^{15} n_{i-1}$$

where the sum need run only from 2 to 15 rather than 1 to 15 since there are no numbers less than 1 (so n_1 must equal 0). Stated more simply, $N = i(p)$ is the number of permutation inversions in the list of numbers. Also define e to be the row number of the empty square.

2	1	3	4
5	6	7	8
9	10	11	12
13	14	15	

Then if $N + e$ is even, the position is possible, otherwise it is not. Stated more simply, if the permutation symbol $(-1)^{i(p)}$ of the list is $+1$, the position is possible, whereas is the signature is -1 , it is not. This can be formally proved using alternating groups. For example, in the arrangement shown above, $n_2 = 1$ (2 precedes 1) and all other $n_i = 0$, so $N = 1$ and the puzzle cannot be solved.

Exercise

9	10	5	4
2	15	14	11
12	13	1	3
	6	8	7

The inversion counts are

8	8	4	3
1	9	8	5
5	5	0	0
	0	1	0

, giving an inversion sum of 57. Since this number is odd, the above arrangement of the puzzle cannot be solved.

“Impossible”

(ในตำแหน่ง $n_1 = 8$ หาได้จากดูจำนวนค่าที่น้อยกว่า 9 ไ้จากตำแหน่งที่พิจารณา ไปจนถึงตำแหน่งสุดท้าย ว่ามีค่าที่ต่ำกว่า 9 กี่ค่า คำตอบที่ได้คือ 8 ค่า ตำแหน่ง $n_{15} = 1$ เนื่องจากมีค่าต่ำกว่า 8 อยู่ 1 ค่า คือ 7 ดังนั้น คำตอบที่ได้คือ 1)